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Semileptonic B_c decays and charmonium distribution amplitude

Tao Huang[a], Fen Zuo[b]

Institute of High Energy Physics, Chinese Academy of Sciences, P.O.Box 918, Beijing 100049, P.R. China

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Abstract. In this paper we study the semileptonic decays of the B_c meson in the light-cone sum rule (LCSR) approach. The result for each channel depends on the corresponding distribution amplitude (DA) of the final meson. For the case of B_c decaying into a pseudoscalar meson, to twist-3 accuracy only the leading twist distribution amplitude is involved if we start from a chiral current. If we choose a suitable chiral current in the vector meson case, the main twist-3 contributions are also eliminated and we can consider the leading twist contribution only. The leading twist distribution amplitudes of the charmonium and other heavy mesons are given by a model approach in a reasonable way. Employing this charmonium distribution amplitude we find a cross section $\sigma(e^+e^- \to J/\psi + \eta_c) \simeq 22.8$ fb that is consistent with Belle and BaBar data. Based on this model, we calculate the form factors for various B_c decay modes in the corresponding regions. Extrapolating the form factors to the whole kinetic regions, we get the decay widths and branching ratios for various B_c decay modes including their τ modes when they are kinematically accessible.

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1 Introduction

The B_c meson has been observed by the CDF and D0 groups in different channels [1–4]. The semileptonic decays of B_c was studied in [5, 6] using the BSW (Bauer, Stech, Wirbel) model [7] and the IGSW (Isgur, Grinstein, Scora, Wise) model [8, 9], and in the frame work of the Bethe–Salpeter equation in [10] and in the relativistic constituent quark model in [11]. Alongside the small differences in the partial decay widths in these models, the first estimates made on the basis of the three-point (3P) QCD sum rules (SR) [12, 13] are significantly smaller. The reason was supposed to be the sizeable role of Coulomb corrections, which implied the summation of α_s/v corrections significant in B_c [14, 15]. It is suggested that the discrepancy observed between the QCD sum rules and the quark models can be eliminated by including these higher QCD corrections.

However, the 3PSR inherits some problems when describing heavy-to-light transitions, the main one being that some of the form factors have a nasty behavior in the heavy quark limit [16]. The reason is that, when almost the whole momentum is carried by one of the constituents, the distribution amplitude (DA) of the final meson cannot be described by a short-distance expansion. Moreover, the calculation for the form factors is valid only at the point $q^2 = 0$, and a pole approximation has to be employed to study the semileptonic decays. These limit the applicability of QCD sum rules based on the short-distance expansion of a three-point correlation function to heavy-to-light transitions, and this calls for an expansion around the light-cone, as realized in the light-cone sum rule approach. In this paper we will try to study the semileptonic decays of the B_c meson in this approach and compare the results with the traditional sum rule approach.

The semileptonic decays of the B_c meson involve the transitions $B_c \to \eta_c$, J/ψ, D, D^*, B, B^*, B_s and B_s^*. For the case of B_c decaying into a pseudoscalar meson, to twist-3 accuracy only the leading twist distribution amplitude is involved if we start from a chiral current. If we choose a suitable chiral current in the vector meson case, the main twist-3 contributions are also eliminated, and we can consider the leading twist contribution only. The result depends on the corresponding distribution amplitude of the final meson. We have to construct realistic models for describing heavy quarkonium and other heavy mesons. In particular, the behavior of the DAs of η_c and J/ψ is an interesting subject by the Belle result for the cross section $\sigma(e^+e^- \to J/\psi + \eta_c)$. Hence we pay more attention to a discussion of the DA of heavy quarkonium. We calculate the cross section $\sigma(e^+e^- \to J/\psi + \eta_c)$ by employing our charmonium distribution amplitude, and the result is consistent with the experimental data. Based on the phenomenological model for the leading twist DA, we calculate the form factors for various B_c decay modes in the corresponding regions. Then we extrapolate the form factors to the whole kinetic regions, and get the decay widths and branching ratios for various B_c decay modes including their τ modes when they are kinematically accessible.

[a] e-mail: huangtao@mail.ihep.ac.cn
[b] e-mail: zuof@mail.ihep.ac.cn

This paper is organized as follows. In the following section we derive the LCSRs for the form factors for various B_c decay modes. A discussion of the DA models for charmonium and other heavy mesons is given in Sect. 3. In Sect. 4 the cross section $\sigma(e^+e^- \to J/\psi + \eta_c)$ is calculated by using our charmonium distribution amplitude. Section 5 is devoted to the numerical results for the semileptonic B_c decays and comparison with other approaches. The last section is reserved for our summary.

2 LCSRs for the B_c semileptonic form factors

According to the definition, the weak transition matrix element $B_c \to P(V)$ can be parametrized in term of the form factors in the following way:

$$\begin{aligned}\langle P(p_2)|\bar{q}\gamma_\mu Q|B_c(p_1)\rangle \\ = f_+(q^2)(p_1+p_2)_\mu + f_-(q^2)q_\mu\,,\end{aligned} \tag{1}$$

$$\begin{aligned}&\langle V(p_2)|\bar{q}\gamma_\mu(1-\gamma_5)Q|B_c(p_1)\rangle \\ &= -\mathrm{i}e^*_\mu(m_{B_c}+m_V)A_1(q^2) + \mathrm{i}(p_1+p_2)_\mu(e^*q)\frac{A_+(q^2)}{m_{B_c}+m_V} \\ &\quad + \mathrm{i}q_\mu(e^*q)\frac{A_-(q^2)}{m_{B_c}+m_V} \\ &\quad + \epsilon_{\mu\alpha\beta\gamma}e^{*\alpha}q^\beta(p_1+p_2)^\gamma\frac{V(q^2)}{m_{B_c}+m_V}\,,\end{aligned} \tag{2}$$

where $q = p_1 - p_2$ is the momentum transfer, and e^*_μ is the polarization vector of the vector meson.

For $B_c \to Pl\tilde{\nu}$, we follow [17] and consider the correlator $\Pi_\mu(p,q)$ with the chiral current,

$$\begin{aligned}&\Pi_\mu(p,q) \\ &= \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx} \\ &\times \langle P(p)\big|T\big\{\bar{q}(x)\gamma_\mu(1+\gamma_5)Q_1(x), \bar{Q}_1(0)\mathrm{i}(1+\gamma_5)Q_2(0)\big\}\big|0\rangle \\ &= \Pi_+(q^2,(p+q)^2)(2p+q)_\mu + \Pi_-(q^2,(p+q)^2)q_\mu\,.\end{aligned} \tag{3}$$

A standard procedure, concentrating on $\Pi_+(q^2,(p+q)^2)$, results in the following LCSR for $f_+(q^2)$:

$$\begin{aligned}f_+(q^2) = &\frac{m_1(m_1+m_2)f_P}{m^2_{B_c}f_{B_c}}\mathrm{e}^{m^2_{B_c}/M^2} \\ &\times \int_{\Delta_P}^1 \mathrm{d}u\frac{\varphi(u)}{u}\exp\left[-\frac{m_1^2-\bar{u}(q^2-um_P^2)}{uM^2}\right] \\ &+ \text{higher twist terms}\,,\end{aligned} \tag{4}$$

with $\bar{u} = 1-u$ and

$$\begin{aligned}\Delta_P = \Big[&\sqrt{\left(s_0^P-q^2-m_P^2\right)^2+4m_P^2\left(m_1^2-q^2\right)} \\ &- \left(s_0^P-q^2-m_P^2\right)\Big]\Big/\left(2m_P^2\right)\,,\end{aligned} \tag{5}$$

where m_1 is the mass of the decay quark Q_1, m_2 the mass of the spectator quark Q_2, and s_0 and M^2 denote the corresponding threshold value and the Borel parameter, respectively. In deriving (4) the following definition of the leading twist distribution amplitude (DA) $\varphi(u)$ of the pseudoscalar meson has been used:

$$\begin{aligned}\langle P(p)|T\bar{q}(x)\gamma_\mu\gamma_5 Q(0)|0\rangle = -\mathrm{i}p_\mu f_P\int_0^1 \mathrm{d}u\mathrm{e}^{\mathrm{i}upx}\varphi(u) \\ + \text{higher twist terms}\,,\end{aligned} \tag{6}$$

with u being the momentum fraction carried by $\bar{q}$. It has been pointed out in [17] that all the twist-3 contributions have been eliminated, so those DAs entering the higher twist terms in (4) are at least of twist 4. By repeating the procedure for $\Pi_-(q^2,(p+q)^2)$, we find a simple relation between $f_+(q^2)$ and $f_-(q^2)$ up to this accuracy:

$$f_-(q^2) = -f_+(q^2)\,. \tag{7}$$

For $B_c \to Vl\tilde{\nu}$ we choose the following correlator as our starting point:

$$\begin{aligned}&\Pi_\mu(p,q) \\ &= -\mathrm{i}\int \mathrm{d}^4x\mathrm{e}^{\mathrm{i}qx} \\ &\times \langle V(p)\big|T\big\{\bar{q}(x)\gamma_\mu(1-\gamma_5)Q_1(x), \bar{Q}_1(0)(1+\gamma_5)Q_2(0)\big\}\big|0\rangle \\ &= \Gamma^1 e^*_\mu - \Gamma^+(e^*q)(2p+q)q_\mu - \Gamma^-(e^*q)q_\mu \\ &\quad + \mathrm{i}\Gamma^V\varepsilon_{\mu\alpha\beta\gamma}e^{*\alpha}q^\beta p^\gamma\,.\end{aligned} \tag{8}$$

Also we take the standard definition of the twist-2 and twist-3 distribution amplitudes of the vector meson (see, e.g., [18]), and neglect higher twist DAs, which are supposed to be less important in comparison with those written below:

$$\begin{aligned}&\langle V(p)|\bar{q}_\beta(x)Q_\alpha(0)|0\rangle \\ &= \frac{1}{4}\int_0^1 \mathrm{d}u\mathrm{e}^{\mathrm{i}upx} \\ &\quad\times\Bigg\{f_V m_V\left[\hat{e}^* g_\perp^{(v)}(u) + \hat{p}\frac{(e^*x)}{(px)}\left(\phi_\parallel(u)-g_\perp^{(v)}(u)\right)\right] \\ &\quad - \mathrm{i}f_V^T\sigma_{\mu\nu}e^{*\mu}p^\nu\phi_\perp(u) \\ &\quad + \frac{m_V}{4}\left(f_V - f_V^T\frac{m_q+M_Q}{m_V}\right)\epsilon^\mu_{\nu\alpha\beta}\gamma_\mu\gamma_5 e^{*\nu}p^\alpha x^\beta g_\perp^{(a)}(u)\Bigg\}_{\alpha\beta}\,,\end{aligned} \tag{9}$$

In (9) u is also the momentum fraction of $\bar{q}$, and $m_q(M_Q)$ is the mass of $\bar{q}(Q)$.

Similarly one can obtain the following sum rules for $A_1(q^2)$, $A_\pm(q^2)$ and $V(q^2)$ in (2):

$$\begin{aligned}A_1(q^2) = &\frac{f_V^T(m_1+m_2)}{f_{B_c}m^2_{B_c}(m_{B_c}+m_V)}\mathrm{e}^{m^2_{B_c}/M^2} \\ &\times\int_{\Delta_V}^1\frac{\mathrm{d}u}{u}\exp\left[-\frac{m_1^2-(1-u)\left(q^2-um_V^2\right)}{uM^2}\right] \\ &\times\frac{m_1^2-q^2+u^2m_V^2}{u}\phi_\perp(u)\,,\end{aligned} \tag{10}$$

$$A_+(q^2) = \frac{f_V^T(m_1+m_2)(m_{B_c}+m_V)}{f_{B_c}m_{B_c}^2}\mathrm{e}^{m_{B_c}^2/M^2}\int_{\Delta_V}^1 \frac{\mathrm{d}u}{u} \times \exp\left[-\frac{m_1^2-(1-u)\left(q^2-um_V^2\right)}{uM^2}\right]\phi_\perp(u)\,, \tag{11}$$

$$A_-(q^2) = -A_+(q^2)\,, \tag{12}$$

$$V(q^2) = A_+(q^2)\,, \tag{13}$$

with

$$\Delta_V = \left[\sqrt{\left(s_0^V-q^2-m_V^2\right)^2+4m_V^2\left(m_1^2-q^2\right)} - \left(s_0^V-q^2-m_V^2\right)\right]/\left(2m_V^2\right)\,, \tag{14}$$

and m_1 is the mass of the decay quark Q_1; m_2 is the mass of the spectator quark Q_2.

3 The distribution amplitudes of charmonium and other heavy mesons

The leading twist distribution amplitude for heavy quarkonium, as defined in the previous section, can be related to the light-cone wave function $\psi_M^f(x,\mathbf{k}_\perp)$ by

$$\varphi_M(x) = \frac{2\sqrt{6}}{f_M}\int\frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\psi_M^f(x,\mathbf{k}_\perp)\,, \tag{15}$$

where f_M is the decay constant. In the non-relativistic case, the distribution amplitude $\varphi_M(x)$ goes to a δ-function, and the peak is at the point $x=1/2$. For heavy quarkonium, η_c, the DA should be wider than a δ-like function, since the c quark is not heavy enough. Of course, it goes to a δ-function as the heavy quark mass $m_c^*\to\infty$.

For the massive quark–antiquark system, [19, 20] provide a good solution of the bound state by solving the Bethe–Salpeter equation with the harmonic oscillator potential in the instantaneous approximation: $\psi_{\mathrm{C.M.}}(q^2) = A\exp(-b^2q^2)$. Then one can apply the Brodsky–Huang–Lepage (BHL) prescription [21–23]:

$$\psi_{\mathrm{C.M.}}(q^2) \leftrightarrow \psi_{\mathrm{LC}}\left(\frac{\mathbf{k}_\perp^2+m_Q^{*2}}{x(1-x)}-M^2\right)\,, \tag{16}$$

and one gets the momentum space LC wave function

$$\psi_M(x,\mathbf{k}_\perp) = A_M\exp\left[-b_M^2\frac{\mathbf{k}_\perp^2+m_Q^{*2}}{x(1-x)}\right]\,, \tag{17}$$

where m_Q^* is the heavy quark mass and M is the mass of the quarkonium. Furthermore, the spin structure of the light-cone wave function should be connected with that of the instant-form wave function by considering the Wigner–Melosh rotation. As a result, the full form of the light-cone wave function should be

$$\psi_M^f(x,\mathbf{k}_\perp) = \chi_M(x,\mathbf{k}_\perp)\psi_M(x,\mathbf{k}_\perp)\,, \tag{18}$$

with the Melosh factor

$$\chi_M(x,\mathbf{k}_\perp) = \frac{m_Q^*}{\sqrt{\mathbf{k}_\perp^2+m_Q^{*2}}}\,. \tag{19}$$

After integrating out $\mathbf{k}_\perp$, the leading twist distribution amplitude of heavy quarkonium becomes

$$\varphi_M^f(x) = \frac{\sqrt{6}A_M m_Q^*}{8\pi^{3/2}f_M b_M}\sqrt{x(1-x)}\left[1-\mathrm{Erf}\left(\frac{b_M m_Q^*}{\sqrt{x(1-x)}}\right)\right]\,, \tag{20}$$

where $\mathrm{Erf}(x) = \frac{2}{\pi}\int_0^x\exp(-t^2)\,\mathrm{d}t$. As $m_Q^*\to\infty$, it is certain that $\varphi_M(x)$ goes to a δ-function. This model of the η_c distribution amplitude has been used to study the large-Q^2 behavior of the η_c–γ and η_b–γ transition form factors in [24]. The parameters A_M and b_M^2 in (17) can be determined completely by two constraints. One constraint is from the leptonic decay constant f_M:

$$\int_0^1\mathrm{d}x\int\frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\chi_M(x,\mathbf{k}_\perp)\psi_M(x,\mathbf{k}_\perp) = \frac{f_M}{2\sqrt{6}}, \tag{21}$$

and the other one from the probability of finding the $|Q\bar{Q}\rangle$ state in heavy quarkonium:

$$\int_0^1\mathrm{d}x\int\frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}|\psi_M(x,\mathbf{k}_\perp)|^2 = P_M\,, \tag{22}$$

with $P_M\simeq 1$ for heavy quarkonium. Taking as input the constituent mass $m_c^*\simeq 1.5\,\mathrm{GeV}$ and the decay constant $f_{\eta_c}\simeq 0.40\,\mathrm{GeV}$,[1] we get the corresponding parameters for η_c:

$$A_{\eta_c} = 128.1\,\mathrm{GeV}\,,\quad b_{\eta_c} = 0.427\,\mathrm{GeV}^{-1}. \tag{23}$$

Then the behavior of the leading twist DA of η_c can be given, and a comparison with the model from the QCD sum rule analysis [25] and the model in [18] is plotted in Fig. 1. The moments of these models are given in Table 1. All the distribution amplitudes and corresponding moments are defined at the soft scale $\mu^*\simeq 1\,\mathrm{GeV}$. However, the appropriate scale μ for the wave functions entering the LC sum rules will be $\mu\simeq m_b$ for b-quark decays and $\mu\simeq m_c$ for c-quark decays. Since μ is not far from μ^*, this scale dependence can be neglected in our calculations for simplicity. From Table 1 it can be found that the moments of the model (20) are similar to those in [18], but much larger than that those in [25]. Obviously the Melosh factor $\chi_M(x,\mathbf{k}_\perp)\to 1$ in the heavy quark limit $m_Q^*\to\infty$. If we neglect this factor and integrate $\mathbf{k}_\perp$ from (17), we get

[1] The value of $f_{J/\psi}$ is taken from the leptonic decay of J/ψ: $\Gamma(J/\psi\to e^+e^-) = (16\pi\alpha^2/27)(|f_{J/\psi}|^2/M_{J/\psi})$, $f_{J/\psi}\simeq 0.41\,\mathrm{GeV}$. The one-loop corrections ($\sim\alpha_s/\pi$) to the ratio $\Gamma(\eta_c\to 2\gamma)/\Gamma(J/\psi\to e^+e^-)$ indicate that f_{η_c} is slightly smaller than $f_{J/\psi}$, and we take $f_{\eta_c}\simeq f_{J/\psi}\simeq 0.40\,\mathrm{GeV}$ as in [18].

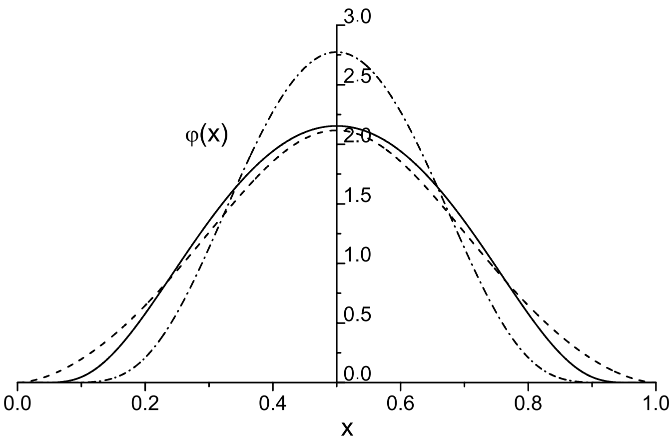

Fig. 1. The model for the leading twist distribution amplitude for η_c (in *solid line*), in comparison with the one in [18] (*dashed line*) and [25] (*dash-dotted line*)

Table 1. The moments of our model for the η_c distribution amplitude, compared with that in [18] and [25]

$\langle\xi^n\rangle$	This work	[18]	[25]
$n=2$	0.21	0.13	0.07
$n=4$	0.053	0.040	0.012
$n=6$	0.018	0.018	0.003

the corresponding distribution amplitude, which has a very simple form:

$$\varphi_M(x) = \frac{\sqrt{3}A_M}{8\pi^2 f_M b_M^2} x(1-x)\exp\left[-\frac{b_M^2 m_Q^{*2}}{x(1-x)}\right]. \quad (24)$$

Actually this is just the wave function proposed in [25] based on a QCD sum rule analysis.

For the vector charmonium, J/ψ, it is expected that the behavior of the transverse distribution amplitude is the same as that of the longitudinal DA, since there is no light quark in the charmonium system, i.e.

$$\phi_{\parallel}(x) = \phi_{\perp}(x) = \varphi^f_{\eta_c}(x)\,, \quad (25)$$

which is confirmed by the moment calculation in the QCD sum rules [26].

For the D, B and B_s mesons, which are composed of one heavy ($\bar{Q}_1$) and one light quark (Q_2), according to the BHL prescription, one takes the following connection:

$$\psi_{\text{C.M.}}(q^2) \leftrightarrow \psi_{\text{LC}}\left(\frac{\mathbf{k}_\perp^2+m_1^{*2}}{x}+\frac{\mathbf{k}_\perp^2+m_2^{*2}}{1-x}-M_P^2\right), \quad (26)$$

with $m_1^*(m_2^*)$ the constituent quark mass of $\bar{Q}_1(Q_2)$, x the momentum fraction carried by $\bar{Q}_1$. Also the Melosh factor should be modified as

$$\chi_P(x,\mathbf{k}_\perp) = \frac{(1-x)m_1^*+xm_2^*}{\sqrt{\mathbf{k}_\perp^2+\left((1-x)m_1^*+xm_2^*\right)^2}}\,. \quad (27)$$

Table 2. Leptonic decay constants (MeV) used in the least-square fit for our model parameters

	This work	Other
f_D	223	$222.6\pm16.7^{+2.8}_{-3.4}$ CLEO [30]
		$201\pm3\pm17$ MILC LAT [31]
		$235\pm8\pm14$ LAT [32]
		$210\pm10^{+17}_{-16}$ UKQCD LAT [33]
		$211\pm14^{+2}_{-12}$ LAT [34]
f_B	190	$216\pm9\pm19\pm4\pm6$ HPQCD LAT [35]
		$177\pm17^{+22}_{-22}$ UKQCD LAT [33]
		$179\pm18^{+34}_{-9}$ LAT [34]
f_{B_s}	220	259 ± 32 HPQCD LAT [35]
		$204\pm16^{+36}_{-0}$ LAT [34]
		$260\pm7\pm26\pm8\pm5$ LAT [36]
		$204\pm12^{+24}_{-23}$ UKQCD LAT [33]

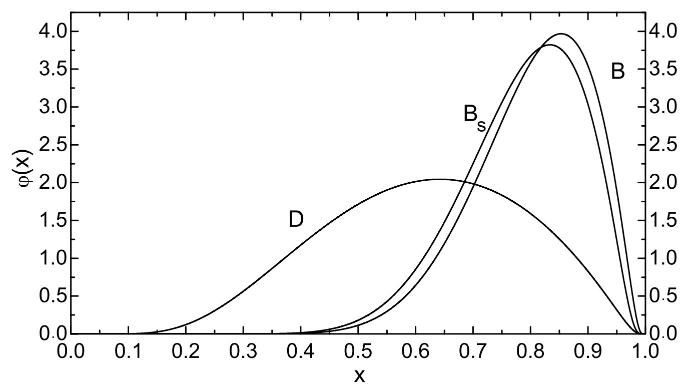

Fig. 2. The leading twist distribution amplitudes for heavy-light pseudoscalar mesons

Then we get the light-cone wave function for the pseudoscalar meson:

$$\psi_P^f(x,\mathbf{k}_\perp) = A_P\chi_P(x,\mathbf{k}_\perp) \times\exp\left[-b_P^2\left(\frac{\mathbf{k}_\perp^2+m_1^{*2}}{x}+\frac{\mathbf{k}_\perp^2+m_2^{*2}}{1-x}\right)\right], \quad (28)$$

and the corresponding distribution amplitude[2] is

$$\varphi_P(x) = \frac{\sqrt{6}A_P y}{8\pi^{3/2}f_P b_P}\sqrt{x(1-x)}\left[1-\text{Erf}\left(\frac{b_P y}{\sqrt{x(1-x)}}\right)\right] \times\exp\left[-b_P^2\frac{\left(xm_2^{*2}+(1-x)m_1^{*2}-y^2\right)}{x(1-x)}\right], \quad (29)$$

where $y = xm_2^*+(1-x)m_1^*$. Similarly, there are two constraints, (21) and (22), to determine the unknown parameters. We take $P_D \simeq 0.8$ and $P_B \simeq P_{B_s} \simeq 1.0$ as suggested in [29]. Taking as input the decay constants (we use the least-square fit values of the results reported by

[2] This model has been used in [27, 28] for the D meson distribution amplitude with different parameters. There was a misprint of the factor $\sqrt{2}$ with the decay constant in [27].

the CLEO Collaboration [30] and lattice simulations [31–36]; see Table 2) and the constituent quark masses $m_u^* = 0.35\,\mathrm{GeV}$, $m_s^* = 0.5\,\mathrm{GeV}$, $m_c^* = 1.5\,\mathrm{GeV}$ and $m_b^* = 4.7\,\mathrm{GeV}$, we get the parameters

$$\begin{aligned} A_D &= 116\,\mathrm{GeV}\,, & b_D &= 0.592\,\mathrm{GeV}^{-1}, \\ A_B &= 1.07\times 10^4\,\mathrm{GeV}\,, & b_B &= 0.496\,\mathrm{GeV}^{-1}, \\ A_{B_s} &= 2.65\times 10^4\,\mathrm{GeV}\,, & b_{B_s} &= 0.473\,\mathrm{GeV}^{-1}. \end{aligned} \tag{30}$$

The distribution amplitudes of these heavy-light mesons are plotted in Fig. 2. The distribution amplitudes of the corresponding vector mesons are treated in the same way as J/ψ.

4 The cross section $\sigma(e^+e^- \to J/\psi + \eta_c)$

Following [18], the cross section $\sigma(e^+e^- \to J/\psi + \eta_c)$ can be calculated by using the distribution amplitudes (20) and (25). We neglect the complicated but slow logarithmic evolution of the wave function forms, and account only for the overall renormalization factors of the local tensor and pseudoscalar currents and for the running of the quark mass, as in [18]. One obtains

$$\sigma(e^+e^- \to J/\psi + \eta_c) \simeq 22.8\,\mathrm{fb}\,, \tag{31}$$

which is consistent with the Belle and BaBar measurements [37, 38] of this cross section:

$$\begin{aligned} &\sigma(e^+e^- \to J/\psi + \eta_c) = 25.6 \pm 2.8 \pm 3.4\,\mathrm{fb} \quad (\mathrm{Belle}), \\ &\sigma(e^+e^- \to J/\psi + \eta_c) = 17.6 \pm 2.8^{+1.5}_{-2.1}\,\mathrm{fb} \quad (\mathrm{BaBar}). \end{aligned} \tag{32}$$

The value given by (31) is of the same order as the numerical result in [18] and much larger than the standard non-perturbative QCD (NRQCD) calculation. The reason is that the DA behavior of the charmonium in our paper and [18] is much wider than a δ-like function due to the relativistic effect. In [25] a similar result ($\sigma \simeq 25.1\,\mathrm{fb}$) was obtained by using a wave function model for charmonium at a scale approximately equal to the momentum running through the gluon propagator. However, since the function form of the model cannot be preserved during the evolution, the result was doubtful. Also our result confirms the observation by [39]. It may be expected that the large disagreement between the experimental data and the standard NRQCD calculation can be resolved by combining the light-cone wave function with the relativistic effect and radiative corrections [40].

5 Numerical result for semileptonic B_c decays

For the decay constant of the B_c meson, we recalculate it in the two-point sum rules using the following correlator:

$$K(q^2) = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx} \langle 0|\bar{c}(x)(1-\gamma_5)b(x), \bar{b}(0)(1+\gamma_5)c(0)|0\rangle\,, \tag{33}$$

for consistency. The calculation is performed to leading order in QCD, since the QCD radiative corrections to the sum rule for the form factors are not taken into account. We also neglect the higher power correction corresponding to the gluon condensates. The value of the threshold parameter s_0 is determined by requiring the experimental value of the mass of B_c to be obtained in the reduced sum rule after taking the derivative of the logarithm of the SR with respect to the inverse of the Borel parameter, $1/M^2$. The quark mass parameters entering our formulas are the one-loop pole masses, for which we use $m_b = 4.7\,\mathrm{GeV}$ and $m_c = 1.3\,\mathrm{GeV}$ (cf. Tables 3 and 4 in the reviews of [41, 42] and references therein). To get the experimental value $m_{B_c} = 6.286\,\mathrm{GeV}$ [3], we find that s_0 should be $s_0 \simeq 42.0\,\mathrm{GeV}^2$, which is smaller than the threshold value taken as input in the ordinary sum rule [43]. This will ensure in some sense that the scalar resonances will make a smaller contribution in our sum rule. The corresponding value of f_{B_c} is $f_{B_c} = 0.189\,\mathrm{GeV}$, which is smaller than that in [43], since we do not include the α_s corrections. The same set of parameters will be used in the LCSRs for the form factor in order to reduce the quark mass dependence. Taking the derivative of the logarithm of the LCSR for the form factors with respect to $1/M^2$, we get a sum rule for the mass of the B_c meson. Requiring this sum rule to be consistent with the experimental value at $q^2 = 0$, we can determine M^2 for each LCSR. This results in $M^2(B_c \to \eta_c) = 25.8\,\mathrm{GeV}^2$, $M^2(B_c \to D) = 11.6\,\mathrm{GeV}^2$, $M^2(B_c \to B) = 112\,\mathrm{GeV}^2$ and $M^2(B_c \to B_s) = 111\,\mathrm{GeV}^2$. It seems that the Borel parameters for $B_c \to B(B_s)$ are somewhat large. However, the LCSRs are quite stable in a large region of the Borel parameter, actually $50\,\mathrm{GeV}^2 < M^2 < 150\,\mathrm{GeV}^2$, and we just use the above value for explicit calculation. For the vector meson we simply use the same M^2 as the corresponding pseudoscalar meson just as we do for the DAs. Also we make the assumption that $f_V^T = f_V = f_P$.

With all the parameters chosen, we can proceed to a calculation of all the form factors involved. The results of the form factors at $q^2 = 0$ are given in Table 3 in comparison with those from other approaches. Notice that in our calculation we always have

$$\begin{aligned} &f_+(q^2) > 0, \quad f_-(q^2) < 0, \quad A_1(q^2) > 0\,, \\ &A_+(q^2) > 0, \quad A_-(q^2) < 0, \quad V(q^2) > 0. \end{aligned} \tag{34}$$

In the 3PSR approach the same relations can be obtained, but only in the case of non-relativistic description for both initial and final meson states, e.g., $B_c \to J/\psi(\eta_c)$. In these decay modes the QM results show the same signature pattern, as can be seen in Table 3.

Our calculations for the form factors are only valid in the limited regions where the operator product expansion (OPE) goes through effectively. For b-quark decays, the LCSR is supposed to be valid in $0 < q^2 < m_b^2 - 2m_b\Lambda_{\mathrm{QCD}} \simeq 15\,\mathrm{GeV}$ and for c-quark decays $0 < q^2 < m_c^2 - 2m_c\Lambda_{\mathrm{QCD}} \simeq 0.4\,\mathrm{GeV}$. It turns out that the calculated form factors can be fitted excellently by the parametrization:

$$F_i(q^2) = \frac{F_i(0)}{1 - a_i q^2/m_{B_c}^2 + b_i\left(q^2/m_{B_c}^2\right)^2}\,. \tag{35}$$

Table 3. The values of the form factors at $q^2=0$ in comparison with the estimates in the three-point sum rule (3PSR) (with the Coulomb corrections included) [14, 15] and in the quark model (QM) [11]

Mode		$f_+(0)$	$f_-(0)$	$A_1(0)$	$A_+(0)$	$A_-(0)$	$V(0)$
	This work	0.87	−0.87	0.75	1.69	−1.69	1.69
$B_c \to \bar{c}c[1S]$	3PSR [14, 15]	0.66	−0.36	0.63	0.69	−1.13	1.03
	QM [11]	0.76	−0.38	0.68	0.66	−1.13	0.96
[3pt]	This work	1.02	−1.02	1.01	9.04	−9.04	9.04
$B_c \to B_s^{(*)}$	3PSR [14, 15]	1.3	−5.8	0.69	−2.34	−21.1	12.9
	QM [11]	−0.61	1.83	−0.33	0.40	10.4	3.25
[3pt]	This work	0.90	−0.90	0.90	7.9	−7.9	7.9
$B_c \to B^{(*)}$	3PSR [14, 15]	1.27	−7.3	0.84	−4.06	−29.0	15.7
	QM [11]	−0.58	2.14	−0.27	0.60	10.8	3.27
[3pt]	This work	0.35	−0.35	0.32	0.57	−0.57	0.57
$B_c \to D^{(*)}$	3PSR [14, 15]	0.32	−0.34	0.43	0.51	−0.83	1.66
	QM [11]	0.69	−0.64	0.56	0.64	−1.17	0.98

Extrapolating the calculated form factors to the whole kinetic region using this parametrization, we can proceed to the calculation of the branching ratios of the semileptonic decays of B_c. The results is shown in Table 4 together with those of other approaches, where we have used the following CKM-matrix elements:

$$V_{cb} = 0.0413\,, \qquad V_{ub} = 0.0037\,,$$
$$V_{cs} = 0.974\,, \qquad V_{cd} = 0.224\,. \tag{36}$$

For the b-quark decay modes in the B_c meson, our results for the branching ratios are much larger than the corresponding results in the 3PSR approach. In these decays the kinetic region is rather large, so the branching ratios slightly depend on the absolute value of the form factors at $q^2=0$. In the LCSR approach, the form factors always increase much faster than the simple pole approximation required in the 3PSR analysis, which accounts for the discrepancy in these decays. For the c-quark decays in the B_c meson, where the kinetic region is narrow enough, our results are roughly consistent with the 3PSR approach.

Table 4. Branching ratios (in %) of semileptonic B_c decays into charmonium ground states, and into charm and bottom meson ground states. For the lifetime of the B_c we take $\tau(B_c) = 0.45$ ps

Mode	This work	3PSR [14, 15]	QM [11]	[10]
$\eta_c e\nu$	1.64	0.75	0.98	0.97
$\eta_c \tau\nu$	0.49	0.23	0.27	–
$J/\psi e\nu$	2.37	1.9	2.30	2.30
$J/\psi \tau\nu$	0.65	0.48	0.59	–
$De\nu$	0.020	0.004	0.018	0.006
$D\tau\nu$	0.015	0.002	0.0094	–
$D^*e\nu$	0.035	0.018	0.034	0.018
$D^*\tau\nu$	0.020	0.008	0.019	–
$Be\nu$	0.21	0.34	0.15	0.16
$B^*e\nu$	0.32	0.58	0.16	0.23
$B_s e\nu$	3.03	4.03	2.00	1.82
$B_s^* e\nu$	4.63	5.06	2.6	3.01

6 Summary

The semileptonic decays of the B_c meson are studied in the light-cone sum rule approach. By using suitable chiral currents, we derive simple sum rules for various form factors, which depend mainly on the leading twist distribution amplitude of the final meson. A model with the harmonic oscillator potential for the light-cone wave function is employed. Special attention is paid to the leading DA of charmonium. It has been found that our model is consistent with the QCD sum rule analysis. Also, the moments are found to be similar to the model proposed in [18]. Based on this model, we calculate the form factors for various B_c decay modes in the corresponding regions. Extrapolating the form factors to the whole kinetic regions, we get the decay widths and branching ratios for all the B_c semileptonic decay modes. For the b-quark decay modes in the B_c meson, where the kinetic regions are quite large, our results for the branching ratios are much larger than the 3PSR results. For the c-quark decays in the B_c meson, they are consistent with each other in general.

It is a crucial point that we could construct a realistic model for the light cone wave function of the charmonium, which is not a non-relativistic subject. Based on the solution of the relativistic Bethe–Salpeter equation in the heavy quark system, we provide a model in this paper by using the BHL prescription, and the behavior of the charmonium DA is much wider than the δ-like function that was employed essentially by the approximation of NRQCD. Thus the cross section $\sigma(e^+e^- \to J/\psi + \eta_c)$ can be enhanced considerably and is about 22.8 fb.

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